Let's not promulgate either Fechner's erroneous algorithm or his unidimensional approach

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Murray several times cites Luce and Edwards's (195ł of Fechner's derivation of Fechner's Law from Wel (sects. 1.1, 1.2, and Note 2). Unfortunately, he see have absorbed our message and he makes the same m cal error where it matters some. Fechner correc Equation 2, $\Delta S(I)/\Delta I = C/I$, as the functional equation ing both his hypothesis of equal subjective jnds and th tion of Weber's Law. To solve for $S(I)$, he proposed rithm – calling it a "mathematical auxiliary princi replacing $\Delta S(I)/\Delta I$ by the differential $dS(I)/dI$. Thus, E is transformed into a (well-understood) differential whose solution is Equation 4. Our point was that algorithm happens on a correct solution when eithe Law or the generalization implicit in Equation 5 holds arrive at the correct solution for any other Weber Falmagne (1985) provides a good discussion of algorithm.

This may seem a bit of esoterica, but the issue reap more significant way in section 1.3.3 where Murray Fechner later considered a reformulation (see Equat the initial problem. To "solve" Equation 10 using algorithm, rewrite it as $\Delta S(I)/\Delta I = CS(I)/I$. Replace t differences by a differential to get $dS(I)/dI = CS(I)/$ equivalent to $d[\log S(I)]/dI = Cd(\log I)/dI$. Integr taking exponentials yields Equation 11. Murray a accept this not only as historically accurate, but also matically correct. About the latter he is wrong. One d

ing feature of the "solution" is that it is independent o of the Weber function. Another worse feature is that 11 simply does not satisfy Equation 10 except when (show this, let $I' = I + \Delta I$, and so $\Delta S(I) = S(I') - S(I)$. As is given by Equation 11,

$$\frac{\Delta S(I)}{S(I)} = \frac{S(I')}{S(I)} - = \left[\frac{I'}{I}\right]^C - 1 = \left[\frac{\Delta I}{I} + 1\right]^C$$

which agrees with Equation 10 when and only whe Considering that 34 years have passed since it was n substituting a differential for a ratio of differences i leads to an incorrect solution, it is sad that this not ve mathematical issue continues to mislead.

A conceptual issue permeates Murray's target art plausible to expect successful measurement of an attrib just one independent variable is manipulated? To m edge, the only case where this approach has succeeded extensive measures of physics, of which mass, length and time are prime examples. Such dimensions i concatenation operation of combining two entities exhibit the attribute in question to form a third entity exhibits the attribute. Classically, such operations we sented numerically as addition, although that choice conventional – multiplication is equally good, as are uum of other (associative and commutative) mathemat ations. No other purely one-dimensional example of fu tal measurement is known, and that was the reas Campbell (1920/1957) and the Ferguson (1940) co largely a creature of Campbell's concerns and on whic bell played a major role, concluded that psychology w ble of fundamental measurement: It has no empirical c tion operations of its own.

If one accepts that measurement is a one-dimensional matter, the committee was right.[1] For a careful contemporary treatment of the one-dimensional approach and its ambiguities, see Narens (submitted). Falmagne (personal communication) points out that if one is willing to deal with choice probabilities rather than orderings, it is possible in principle to construct a binary operation over the probability space. The difficulty is that because most choice probabilities are 0 or 1, one is forced to piece together the global scale from highly local data.

What the Ferguson committee failed to acknowledge, and many psychophysical scalers seem to continue to ignore, is that something very like one-dimensional measurement becomes feasible when two or more independent variables affect the same attribute. One can use the resulting trade-off between the independent variables as a source of measurement of the attribute and how the factors combine. Indeed, trade-offs typically induce mathematical concatenation operations on the components. This was completely familiar in physical measurement of such quantities as momentum, energy, density, and so on, but it had not been axiomatized in a fashion analogous to the turn-of-the-century axiomatizations of extensive measurement. The lacuna was corrected in the early 1960s (see Chapter 6 of Krantz et al. 1971 and Chapter 19 of Luce et al. 1990 for historical details).

Thoroughgoing behavioral examples of the trade-off approach are Luce's (1977) axiomatization of power and logarithmic functions when the data are orderings of stimulus pairs, such as loudness to binaural tones; the functional measurement procedures advocated and applied to psychophysical as well as other psychological problems by Anderson (1981; 1982); and the entire complex literature on axiomatizations of preferences and/or judgments of uncertain alternatives. In each case, one uses the trade-off between variables to establish simultaneously the measures involved and the law relating them.

If axiomatics are not to one's taste, another approach is to increase the dependent variables from just choice and/or judgments about psychophysical stimuli to include the time taken to

respond. Here the models are much more process oriented and have little in common with the measurement-theoretic models. The measures of sensation are parameters of the model and are estimated from behavioral data. Again, trade-offs are the name of the game, in these cases often between two or more sensory variables such as signal duration and signal intensity but also between a sensory variable and some sort of motivational criterion. The models simultaneously develop measurement and theory. Summaries of various of these models can be found in Luce (1986) and Townsend and Ashby (1983).

My only point is that one is probably misguided to continue to fuss at the one-dimensional measurement case; unless a relevant concatenation operation or some other rich internal structure can be found, the situation is simply too underdetermined to be of much theoretical interest.

NOTE

1. At the time, the only cases that were understood involved operations with additive representations. Later work (see Chs. 19 and 20 of Luce et al., 1990, for a summary) yields a whole family of inherently nonadditive operations. Homogeneity, described in Luce and Narens (1987), underlies this result; this paper, which is purely expository, gives references to the original contributions.